Adaptive Decision Feedback Reduced-Rank Equalization Based on Joint Iterative Optimization of Adaptive Estimation Algorithms for Multi-Antenna Systems

Rodrigo C. de Lamare †, Are Hjørungnes ‡ and Raimundo Sampaio-Neto §
†*Communication Research Group, Department of Electronics, The University of York, UK*
‡*UniK - University Graduate Center, University of Oslo, Norway*
§*CETUC/Pontifical Catholic University of Rio de Janeiro (PUC-RIO), Brazil*
E-mails: rcdl500@ohm.york.ac.uk, arehj@unik.no, raimundo@cetuc.puc-rio.br

***Abstract*—This paper presents a novel adaptive reduced-rank multi-input-multi-output (MIMO) decision feedback equalization structure based on joint iterative optimization of adaptive estimators. The novel reduced-rank equalization structure consists of a joint iterative optimization of two equalization stages, namely, a projection matrix that performs dimensionality reduction and a reduced-rank estimator that retrieves the desired transmitted symbol. The proposed reduced-rank structure is followed by a decision feedback scheme that is responsible for cancelling the inter-antenna interference caused by the associated data streams. We describe least squares (LS) expressions for the design of the projection matrix and the reduced-rank estimator along with computationally efficient recursive least squares (RLS) adaptive estimation algorithms. Simulations for a MIMO equalization application show that the proposed scheme outperforms the state-of-the-art reduced-rank and the conventional estimation algorithms at about the same complexity.**

***Index Terms*—MIMO systems, equalization, parameter estimation, reduced-rank schemes.**

I. Introduction

The high demand for performance and capacity in wireless networks has led to the development of numerous signal processing and communications techniques for employing the resources efficiently. Recent results on information theory have shown that it is possible to achieve high spectral efficiency [1] and to make wireless links more reliable [2], [3] through the deployment of multiple antennas at both transmitter and receiver. In MIMO communications systems, the received signal is composed by the sum of several transmitted signals which share the propagation environment and are subject to multiple propagation paths and noise at the receiver. The multipath channel originates intersymbol interference (ISI), whereas the non-orthogonality among the signals transmitted gives rise to multi-access interference (MAI) at the receiver.

In order to mitigate the detrimental effects of ISI and MAI, that reduce the performance and the capacity of MIMO systems, the designer has to construct a space-time and MIMO equalizer. The optimal MIMO equalizer known as the maximum likelihood sequence estimation (MLSE) receiver was originally developed in the context of multiuser detection in [4]. However, the exponential complexity of the optimal MIMO equalizer makes its implementation costly for multipath channels with severe ISI and MIMO systems with many antennas. In practice, designers often prefer the deployment of low-complexity MIMO receivers such as the linear [5], [6] and MIMO decision feedback equalizers (DFE) [7], [8]. Among these, the DFE [7], [8] can offer substantially better performance than their linear counterparts due to the interference cancellation capabilities of the feedback section for good channel conditions. These receivers require the estimation of the coefficients used for combining the received data and extracting the desired transmitted symbols. A challenging problem which remains unsolved by conventional estimation techniques is that when the number of elements in the estimator is large, the algorithm requires substantial training for the MIMO DFE and a large number of received symbols to reach its steady-state behavior.

Reduced-rank estimation [17]-[22] is a very powerful and effective technique in low-sample-support situations and in problems with high-order estimators. The advantages of reduced-rank estimators are their faster convergence speed and better tracking performance than existing techniques when dealing with large number of weights. Several reduced-rank methods and systems have been proposed in the last several years, namely, eigen-decomposition techniques [18], the multistage Wiener filter (MWF) [19], and the auxiliary vector filtering (AVF) algorithm [21]. Prior work on reduced-rank estimators for MIMO systems is extremely limited and relatively unexplored, being the work of *Sun et al.* [28] one of the few existing ones in the area.

In this work, we propose a novel adaptive MIMO decision feedback equalization structure based on a novel reduced-rank estimation method. The proposed reduced-rank equalization structure consists of a joint iterative optimization of two equalization stages, namely, a projection matrix that performs dimensionality reduction and a reduced-rank estimator that retrieves the desired transmitted symbol and is then followed by a feedback section that is responsible for cancelling the multi-access interference caused by the associated users. The essence of the proposed scheme is to change the role of the equalization filters of the feedforward section of the MIMO DFE. The projection matrix is responsible for performing dimensionality reduction, whereas the reduced-rank estimator effectively retrieves the desired signal. In order to estimate

the coefficients of the proposed MIMO reduced-rank DFE, we describe least squares (LS) expressions for the design of the projection matrix and the reduced-rank estimator along with computationally efficient recursive least squares (RLS) adaptive estimation algorithms. The performance of the proposed adaptive MIMO reduced-rank DFE and estimation algorithm is assessed and compared with the best known estimation schemes via numerical simulations.

The rest of this paper is structured as follows: The MIMO system model is given in Section II. The conventional adaptive MIMO DFE is reviewed in Section III, whereas the proposed adaptive MIMO reduced-rank DFE is introduced in Section IV. Section V is devoted to the development of the LS estimators and the computationally efficient RLS algorithms. Section VI presents and discusses the simulation results and Section VII gives the concluding remarks of this work.

II. MIMO System Model

Consider a MIMO system with N_T antennas at the transmitter and N_R antennas at the receiver in a spatial multiplexing configuration, as shown in Fig. 1. The signals are transmitted from N_T antennas over multipath channels with L_p propagation paths and are received by N_R antennas. We assume that the channel is constant during each packet transmission and the receiver is synchronized with the main path.

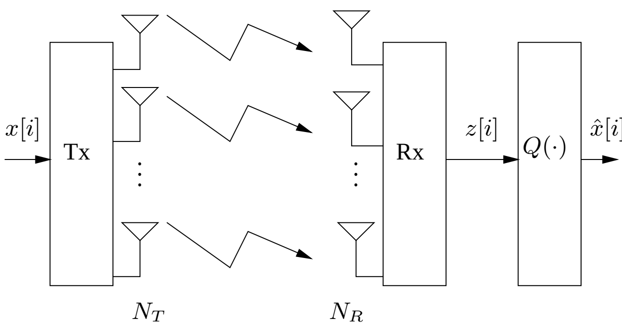

Fig. 1. MIMO system model.

The received signals are filtered by a matched filter, sampled at symbol rate, organized in a window of L symbols ($L > L_p$) for each antenna element and yield the $LN_R \times 1$ received vector

$$\mathbf{y}[i] = \mathbf{H}[i]\mathbf{x}_T[i] + \mathbf{n}[i], \quad (1)$$

where $\mathbf{y}[i] = \big[\mathbf{y}_1^T[i]\ \mathbf{y}_2^T[i]\ \ldots\ \mathbf{y}_{N_R}^T[i]\big]^T$ contains the signals collected by the N_R antennas, the $L \times 1$ vector $\mathbf{y}_k[i] = \big[y_{k,1}[i]\ y_{k,2}[i]\ \ldots\ y_{k,L}[i]\big]^T$, for $k = 1, \ \ldots, \ N_R$, contains the signals collected by the kth antenna and are organized into a vector. The $LN_R \times LN_T$ MIMO channel matrix $\mathbf{H}[i]$ is described by

$$\mathbf{H}[i] = \begin{bmatrix} \mathbf{H}_{1,1}[i] & \mathbf{H}_{1,2}[i] & \ldots & \mathbf{H}_{1,N_T}[i] \\ \mathbf{H}_{2,1}[i] & \mathbf{H}_{2,2}[i] & \ldots & \mathbf{H}_{2,N_T}[i] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{N_R,1}[i] & \mathbf{H}_{N_R,2}[i] & \ldots & \mathbf{H}_{N_R,N_T}[i] \end{bmatrix}, \quad (2)$$

where the $L \times L$ matrix $\mathbf{H}_{k,j}[i]$ describes the multipath channel from antenna k to antenna j. The $LN_T \times 1$ vector $\mathbf{x}_T[i] = \big[\mathbf{x}_1^T[i]\ \mathbf{x}_2^T[i]\ \ldots\ \mathbf{x}_{N_R}^T[i]\big]^T$ is composed by the data symbols transmitted from the N_T antennas at the transmitter with $\mathbf{x}_k[i]$ being the ith transmitted block with dimensions $L \times 1$. The $LN_R \times 1$ vector $\mathbf{n}[i]$ is a complex Gaussian noise vector with zero mean and $E\big[\mathbf{n}[i]\mathbf{n}^H[i]\big] = \sigma^2\mathbf{I}$, where $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively, and $E[\cdot]$ stands for expected value.

III. Adaptive MIMO Decision Feedback Equalizer

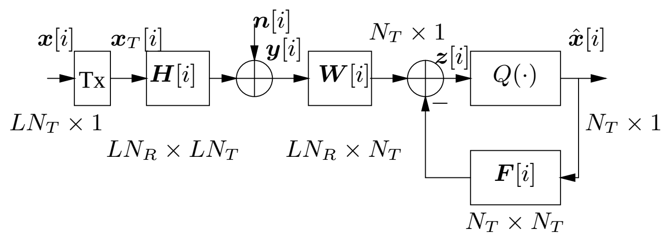

Fig. 2. MIMO system model with conventional decision feedback equalizer.

The conventional MIMO decision feedback equalizer design corresponds to determining an $LN_R \times N_T$ estimator $\mathbf{W}[i] = \big[\mathbf{w}_1[i]\mathbf{w}_2[i]\ldots\mathbf{w}_{N_T}[i]\big]$ that linearly combines the received signal $\mathbf{y}[i]$ and an $N_T \times N_T$ estimator $\mathbf{F}[i] = \big[\mathbf{f}_1[i]\ \mathbf{f}_2[i]\ \ldots\ \mathbf{f}_{N_T}[i]\big]$ that cancel the associated MAI created by the different data streams and the ISI caused by the multipath channels. The block diagram shown in Fig. 2 illustrates how the MIMO DFE works. The estimate $\mathbf{z}[i]$ of the desired symbols is given by

$$\mathbf{z}[i] = \mathbf{W}^H[i]\mathbf{y}[i] - \mathbf{F}^H[i]\hat{\mathbf{x}}[i], \quad (3)$$

where $\hat{\mathbf{x}}[i] = Q\big(\mathbf{W}^H[i]\mathbf{y}[i]\big)$ is the initial decision vector taken with the feedforward section $\mathbf{W}[i]$ and $Q(\ \cdot\ )$ represents a decision device. For the design of the feedback section, we constrain $\mathbf{F}[i]$ to be full and to have zeros along the diagonal to avoid cancelling the desired symbols. This corresponds to parallel decision feedback [28], [29]. Specifically, the non-zero part of the filter $\mathbf{F}[i]$ corresponds to the number of used feedback connections and to the users to be cancelled. The feedback connections used and their associated number of non-zero filter coefficients in $\mathbf{F}[i]$ are equal to $N_T - 1$ for all users.

The detected symbols of the DFE after the interference cancellation $\hat{\mathbf{x}}^{(f)}[i]$ carried out by the feedback section are given by

$$\hat{\mathbf{x}}^{(f)}[i] = Q\big(\mathbf{z}[i]\big). \quad (4)$$

In order to design the LN_R-dimensional estimators $\mathbf{w}_j[i]$ ($j = 1, \ldots, N_T$) that form $\mathbf{W}[i]$, one can resort to stochastic gradient or LS algorithms, as in [6], [7], [8].

IV. Proposed Adaptive MIMO Reduced-Rank Decision Feedback Equalizer

In the proposed reduced-rank linear MIMO equalizer, the signal processing tasks are carried out in two stages, as

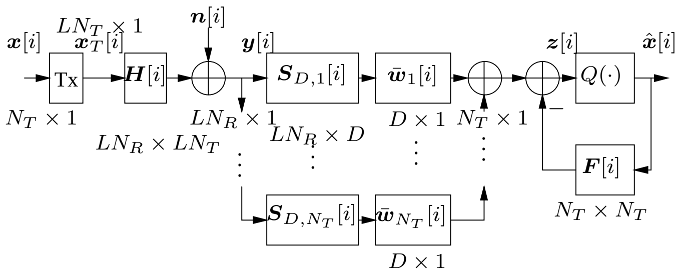

Fig. 3. Proposed MIMO reduced-rank decision feedback equalizer.

illustrated in Fig. 3. Firstly, we consider the dimensionality reduction of $\mathbf{y}[i]$ by projecting the received vector onto a lower dimensional subspace. Specifically, consider an $LN_R \times D$ projection matrix $\mathbf{S}_{D,j}[i]$ which carries out a dimensionality reduction on the received data for extracting the symbols transmitted from antenna j as given by

$$\bar{\mathbf{y}}_j[i] = \mathbf{S}_{D,j}^H[i]\mathbf{y}[i], \tag{5}$$

where, in what follows, all D-dimensional quantities are denoted with a "bar". The resulting projected received vector $\bar{\mathbf{y}}_j[i]$ is the input to an estimator represented by the $D \times 1$ vector $\bar{\mathbf{w}}_j[i] = [\bar{w}_{j,1}[i] \ \bar{w}_{j,2}[i] \ \ldots \bar{w}_{j,D}[i]]^T$ for time interval i. The reduced-rank estimator output corresponding to the ith time instant and estimators $\bar{\mathbf{w}}_j[i]$, $\mathbf{S}_{D,j}[i]$, and $\mathbf{f}_j[i]$ for extracting the symbol transmitted from antenna j is

$$\begin{aligned} z_j[i] &= \bar{\mathbf{w}}_j^H[i]\mathbf{S}_{D,j}^H[i]\mathbf{y}[i] - \mathbf{f}_j^H[i]\hat{\mathbf{x}}[i] \\ &= \bar{\mathbf{w}}_j^H[i]\bar{\mathbf{y}}[i] - \mathbf{f}_j^H[i]\hat{\mathbf{x}}[i], \end{aligned} \tag{6}$$

where the feedback section estimators form $\mathbf{F}[i] = \big[\mathbf{f}_1[i] \ \ldots \ \mathbf{f}_{N_T}[i]\big]$, which is constrained to have zeros along the main diagonal to avoid cancelling the desired symbols, similarly to the conventional MIMO DFE. We will consider here the case where the number of transmit antennas N_T is reasonably small which leads to small estimators in $\mathbf{F}[i]$. From the above outputs, we construct the vector $\mathbf{z}[i] = \big[z_1[i] \ldots z_j[i] \ldots z_{N_T}[i]\big]^T$. The detected symbols of the proposed reduced-rank MIMO DFE after the interference cancellation $\hat{\mathbf{x}}^{(f)}[i]$ are obtained with

$$\hat{\mathbf{x}}^{(f)}[i] = Q\big(\mathbf{z}[i]\big). \tag{7}$$

We remark that when N_T becomes large and consequently the number of used feedback connections corresponding to the data streams to be cancelled, the designer can incorporate this in a single large matrix filter $\mathbf{C}^T[i] = \big[\mathbf{W}^T[i]\mathbf{F}^T[i]\big] = \big[\mathbf{c}_1^T[i] \ \ldots \ \mathbf{c}_{N_T}^T\big]$ and stack the vectors $\mathbf{y}[i]$ and $\hat{\mathbf{x}}[i]$, forming the $(LN_R + N + T) \times 1$ vector $\mathbf{a}^T[i] = \big[\mathbf{y}^T[i] \ \ \hat{\mathbf{x}}^T[i]\big]$. In this case, the output of the MIMO DFE would be $z_j[i] = \mathbf{c}_j^H[i]\tilde{\mathbf{S}}_{D,j}^H[i]\mathbf{a}[i]$, where $\tilde{\mathbf{S}}_{D,j}[i]$ is a modified projection matrix with dimensions $(LN_R + N + T) \times D$.

V. Proposed Least Squares Design and Reduced-Rank RLS Algorithms

In this section, we present a joint iterative exponentially weighted least squares (LS) estimator design of the parameters $\mathbf{S}_{D,j}[i]$, $\mathbf{w}_j[i]$, and $\mathbf{f}_j[i]$ of the proposed MIMO reduced-rank DFE and a computationally efficient RLS algorithm for implementing the proposed LS estimator.

A. Least Squares Design

In order to design $\mathbf{S}_{D,j}[i]$, $\bar{\mathbf{w}}_j[i]$, and $\mathbf{f}_j[i]$, we describe a joint iterative LS optimization algorithm. Let us consider the exponentially-weighted LS expressions for the estimators $\mathbf{S}_{D,j}[i]$, $\bar{\mathbf{w}}_j[i]$, and $\mathbf{f}_j[i]$ can be computed via the cost function

$$\mathcal{C} = \sum_{l=1}^{i} \lambda^{i-l}\big|x_j[l] - \bar{\mathbf{w}}_j^H[i]\mathbf{S}_{D,j}^H[i]\mathbf{y}[l] + \mathbf{f}_j^H[i]\hat{\mathbf{x}}[l]\big|^2, \tag{8}$$

where λ is the forgetting factor.

By minimizing (8) with respect to $\mathbf{S}_{D,j}[i]$, we obtain

$$\mathbf{S}_{D,j}[i] = \mathbf{R}^{-1}[i]\Big(\mathbf{P}_{D,j}[i] + \mathbf{P}_{\mathbf{f}_j}[i]\Big)\mathbf{R}_{\bar{\mathbf{w}}_j}^{-1}[i], \tag{9}$$

where $\mathbf{P}_{D,j}[i] = \sum_{l=1}^{i} \lambda^{i-l}x_j^*[l]\mathbf{y}[l]\mathbf{w}_j^H[i]$, $\mathbf{R}[i] = \sum_{l=1}^{i} \lambda^{i-l}\mathbf{y}[l]\mathbf{y}^H[l]$, $\mathbf{P}_{\mathbf{f}_j}[i] = \sum_{l=1}^{i} \lambda^{i-l}\mathbf{y}[l]\mathbf{w}_j^H[i]\hat{\mathbf{x}}^H[l]$, and $\mathbf{R}_{w_j}[i] = \sum_{l=1}^{i} \lambda^{i-l}\mathbf{w}_j[l]\mathbf{w}_j^H[l]$. By minimizing (8) with respect to $\bar{\mathbf{w}}_j[i]$, the reduced-rank estimator becomes

$$\bar{\mathbf{w}}_j[i] = \bar{\mathbf{R}}_j^{-1}[i]\Big(\bar{\mathbf{p}}_j[i] + \mathbf{D}_j[i]\mathbf{f}_j[i]\Big), \tag{10}$$

where $\bar{\mathbf{p}}_j[i] = \mathbf{S}_{D,j}^H[i]\sum_{l=1}^{i} \lambda^{i-l}x_j^*[l]\mathbf{y}[l] = \sum_{l=1}^{i} \lambda^{i-l}x_j^*[l]\bar{\mathbf{y}}[l]]$, the reduced-rank estimated correlation matrix is $\bar{\mathbf{R}}_j[i] = \mathbf{S}_{D,j}^H[i]\sum_{l=1}^{i} \lambda^{i-l}\mathbf{y}[l]\mathbf{y}^H[l]\mathbf{S}_{D,j}[i]$ and $\mathbf{D}_j[i] = \mathbf{S}_{D,j}^H[i]\sum_{l=1}^{i} \lambda^{i-l}\mathbf{y}[l]\hat{\mathbf{x}}^H[l]$. By minimizing (8) with respect to $\mathbf{f}_j[i]$ we obtain

$$\mathbf{f}_j[i] = \mathbf{B}^{-1}[i]\Big(\mathbf{D}_j^H[i]\bar{\mathbf{w}}_j[i] - \mathbf{b}[i]\Big), \tag{11}$$

where $\mathbf{B}[i] = \sum_{l=1}^{i} \lambda^{i-l}\hat{\mathbf{x}}[l]\hat{\mathbf{x}}^H[l]$ and $\mathbf{b}[i] = \sum_{l=1}^{i} \lambda^{i-l}x_j^*[l]\hat{\mathbf{x}}[l]$. The associated sum of error squares (SES) is

$$\begin{aligned} \text{SES} = \sigma_x^2 &- \bar{\mathbf{w}}_j^H[i]\mathbf{S}_{D,j}^H[i]\mathbf{p}[i] - \mathbf{p}^H[i]\mathbf{S}_{D,j}[i]\bar{\mathbf{w}}_j[i] \\ &+ \bar{\mathbf{w}}^H[i]\mathbf{S}_{D,j}^H[i]\mathbf{R}\mathbf{S}_{D,j}[i]\bar{\mathbf{w}}_j[i] \\ &- \mathbf{f}_j^H[i]\mathbf{D}_j^H[i]\mathbf{S}_{D,j}[i]\bar{\mathbf{w}}_j[i] \\ &- \bar{\mathbf{w}}^H[i]\mathbf{S}_{D,j}[i]\mathbf{D}_j[i]\mathbf{f}_j[i] + \mathbf{f}_j^H\mathbf{b}[i] \\ &+ \mathbf{b}^[i]\mathbf{f}_j[i] + \mathbf{f}_j^H[i]\mathbf{B}[i]\mathbf{f}_j[i], \end{aligned} \tag{12}$$

where $\sigma_x^2 = \sum_{l=1}^{i} \lambda^{i-l}|x[l]|^2$. Note that the expressions in (9), (10), and (11) are not closed-form solutions for $\bar{\mathbf{w}}_j[i]$, $\mathbf{S}_{D,j}[i]$, and $\mathbf{f}_j[i]$ since they depend on each other and, thus, they have to be iterated with an initial guess to obtain a solution. The key strategy lies in the joint optimization of the estimators. The rank D must be set by the designer to ensure appropriate performance. The computational complexity of implementing (9), (10), and (11) is cubic with the number of elements in the estimators, namely, LN_R, D, and N_T, respectively. In what follows, we introduce efficient RLS algorithms for implementing the estimators with a quadratic cost.

B. Reduced-Rank Recursive Least Squares Algorithms

In this part, we present RLS algorithms for efficiently implementing the LS design of the previous subsection. Firstly, let us define $\mathbf{P}[i] = \mathbf{R}^{-1}[i]$, $\mathbf{Q}_{\bar{\mathbf{w}}_j}[i-1] = \mathbf{P}_{\bar{\mathbf{w}}_j}^{-1}[i]$, $\mathbf{P}_{D,j}[i] = \lambda \mathbf{P}_D[i-1] + x_j^*[i]\mathbf{y}[i]\mathbf{w}^H[i]$, $\mathbf{P}_{f,j}[i] = \lambda \mathbf{P}_{f,j}[i-1] + x_j^*[i]\mathbf{y}[i]\bar{\mathbf{w}}^H[i]$, and rewrite the expression in (9) as follows

$$\begin{aligned} \mathbf{S}_{D,j}[i] &= \mathbf{R}^{-1}[i]\big(\mathbf{P}_{D,j}[i] + \mathbf{P}_{\mathbf{f}_j}[i]\big)\mathbf{P}_{\bar{\mathbf{w}}_j}[i] \\ &= \mathbf{P}[i]\big(\mathbf{P}_{D,j}[i] + \mathbf{P}_{\mathbf{f}_j}[i]\big)\mathbf{Q}_{\bar{\mathbf{w}}_j}[i-1] \\ &= \mathbf{S}_{D,j}[i-1] + \mathbf{k}[i]\big(x_j^*[i]\mathbf{t}_j^H[i] \\ &\quad - \mathbf{y}^H[i]\mathbf{S}_{D,j}[i-1] + \mathbf{t}_j^H[i]\hat{\mathbf{x}}^H[i]\mathbf{f}_j[i]\big). \end{aligned} \tag{13}$$

By defining the vector $\mathbf{t}_j[i] = \mathbf{Q}_{\bar{\mathbf{w}}_j}[i]\bar{\mathbf{w}}_j[i]$ and using the fact that $\bar{\mathbf{y}}^H[i-1] = \mathbf{y}^H[i-1]\mathbf{S}_{D,j}[i-1]$ we arrive at

$$\begin{aligned} \mathbf{S}_{D,j}[i] &= \mathbf{S}_{D,j}[i-1] - \mathbf{k}[i]\big(x_j^*[i]\mathbf{t}^H[i] \\ &\quad - \mathbf{y}^H[i]\mathbf{S}_{D,j}[i-1] + \mathbf{t}_j^H[i]\hat{\mathbf{x}}^H[i]\mathbf{f}_j[i]\big), \end{aligned} \tag{14}$$

where the Kalman gain vector for the estimation of $\mathbf{S}_{D,j}[i]$ is

$$\mathbf{k}[i] = \frac{\lambda^{-1}\mathbf{P}[i-1]\mathbf{y}[i]}{1 + \lambda^{-1}\mathbf{y}^H[i]\mathbf{P}[i-1]\mathbf{y}[i]}, \tag{15}$$

and the update for the matrix $\mathbf{P}[i]$ employs the matrix inversion lemma

$$\mathbf{P}[i] = \lambda^{-1}\mathbf{P}[i-1] - \lambda^{-1}\mathbf{k}[i]\mathbf{y}^H[i]\mathbf{P}[i-1], \tag{16}$$

the vector $\mathbf{t}_j[i]$ is updated as follows

$$\mathbf{t}_j[i] = \frac{\lambda^{-1}\mathbf{Q}_{\bar{\mathbf{w}}_j}[i-1]\bar{\mathbf{w}}_j[i-1]}{1 + \lambda^{-1}\bar{\mathbf{w}}_j^H[i-1]\mathbf{Q}_{\bar{\mathbf{w}}_j}[i-1]\bar{\mathbf{w}}_j[i-1]}, \tag{17}$$

and the matrix inversion lemma is used to update $\mathbf{Q}_{\bar{\mathbf{w}}_j}[i]$ as described by

$$\mathbf{Q}_{\bar{\mathbf{w}}_j}[i] = \lambda^{-1}\mathbf{Q}_{\bar{\mathbf{w}}_j}[i-1] - \lambda^{-1}\mathbf{t}_j[i]\bar{\mathbf{w}}_j^H[i-1]. \tag{18}$$

Equations (13)-(18) constitute the part of the proposed RLS algorithms for estimating the projection matrix $\mathbf{S}_{D,j}[i]$. In order to develop the second part of the algorithm that estimates $\bar{\mathbf{w}}_j[i]$, let us consider the expression in (10) with its associated quantities, i.e., the matrix $\bar{\mathbf{R}}_j[i] = \sum_{l=1}^{i}\lambda^{i-l}\bar{\mathbf{y}}[l]\bar{\mathbf{y}}^H[l]$, the vector $\bar{\mathbf{p}}_j[i] = \sum_{l=1}^{i}\lambda^{i-l}x_j^*[l]\bar{\mathbf{y}}[l]$ and the matrix $\mathbf{D}_j[i] = \sum_{l=1}^{i}\lambda^{i-l}\bar{\mathbf{y}}[l]\hat{\mathbf{x}}^H[l]$. Let us define $\bar{\boldsymbol{\Phi}}_j[i] = \mathbf{R}_j^{-1}[i]$ and rewrite $\bar{\mathbf{p}}_j[i]$ as $\bar{\mathbf{p}}_j[i] = \lambda\bar{\mathbf{p}}_j[i-1] + x_j^*[i]\bar{\mathbf{y}}_j[i]$ and $\mathbf{D}_j[i]$ as $\mathbf{D}_j[i] = \mathbf{D}_j[i-1] + \bar{\mathbf{y}}[i]\hat{\mathbf{x}}^H[i]$. We can write (10) as follows

$$\begin{aligned} \bar{\mathbf{w}}_j[i] &= \bar{\boldsymbol{\Phi}}_j[i]\big(\bar{\mathbf{p}}_j[i] + \mathbf{D}_j[i]\mathbf{f}_j[i]\big) \\ &= \bar{\mathbf{w}}_j[i-1] - \bar{\mathbf{k}}_j[i]\bar{\mathbf{y}}_j^H[i]\bar{\mathbf{w}}_j[i-1] + \bar{\mathbf{k}}_j[i]x_j^*[i] \\ &\quad + \hat{\mathbf{x}}^H[i]\mathbf{f}_j[i] \\ &= \bar{\mathbf{w}}[i-1] + \bar{\mathbf{k}}_j[i]\big(x_j^*[i] - \bar{\mathbf{y}}_j^H[i]\bar{\mathbf{w}}_j[i-1] + \hat{\mathbf{x}}^H[i]\mathbf{f}_j[i]\big). \end{aligned} \tag{19}$$

By defining $\xi_j[i] = x_j[i] - \bar{\mathbf{w}}_j^H[i-1]\bar{\mathbf{y}}_j[i] + \mathbf{f}_j^H[i]\hat{\mathbf{x}}[i]$ we arrive at the proposed RLS algorithm for estimating $\bar{\mathbf{w}}_j[i]$

$$\bar{\mathbf{w}}_j[i] = \bar{\mathbf{w}}_j[i-1] + \bar{\mathbf{k}}_j[i]\xi_j^*[i], \tag{20}$$

where the Kalman gain vector is given by

$$\bar{\mathbf{k}}_j[i] = \frac{\lambda^{-1}\bar{\boldsymbol{\Phi}}_j[i-1]\bar{\mathbf{y}}_j[i]}{1 + \lambda^{-1}\bar{\mathbf{y}}_j^H[i]\bar{\boldsymbol{\Phi}}_j[i-1]\bar{\mathbf{y}}_j[i]}, \tag{21}$$

and the update for the matrix inverse $\bar{\boldsymbol{\Phi}}[i]$ employs the matrix inversion lemma

$$\bar{\boldsymbol{\Phi}}_j[i] = \lambda^{-1}\bar{\boldsymbol{\Phi}}_j[i-1] - \lambda^{-1}\bar{\mathbf{k}}_j[i]\bar{\mathbf{y}}_j^H[i]\bar{\boldsymbol{\Phi}}_j[i-1]. \tag{22}$$

The third part of the proposed algorithm estimates the feedback filter section $\mathbf{f}_j[i]$ and uses the expression in (11) in a similar way to the development of the estimation procedure of $\bar{\mathbf{w}}_j[i]$. Let us define $\mathbf{P}_B[i] = \mathbf{B}^{-1}[i]$ and rewrite $\mathbf{b}[i]$ as $\mathbf{b}[i] = \lambda\mathbf{b}[i-1] + x_j^*[i]\bar{\mathbf{x}}[i]$. Then, let us express (11) in an alternative way as follows

$$\begin{aligned} \mathbf{f}_j[i] &= \mathbf{P}_B[i]\big(\mathbf{D}_j^H[i]\bar{\mathbf{w}}_j[i] - \mathbf{b}[i]\big) \\ &= \mathbf{f}_j[i-1] - \mathbf{k}_{B,j}[i]\big(x_j^*[i] - \bar{\mathbf{y}}_j^H[i]\bar{\mathbf{w}}_j[i] + \hat{\mathbf{x}}^H[i]\mathbf{f}_j[i-1]\big) \\ &= \mathbf{f}[i-1] - \mathbf{k}_{B,j}[i]\xi_k^*[i], \end{aligned} \tag{23}$$

where the Kalman gain vector $\mathbf{k}_B[i]$ is given by

$$\mathbf{k}_B[i] = \frac{\lambda^{-1}\mathbf{P}_B[i-1]\hat{\mathbf{x}}[i]}{1 + \lambda^{-1}\hat{\mathbf{x}}^H[i]\mathbf{P}_B[i-1]\hat{\mathbf{x}}[i]}, \tag{24}$$

and the update for the matrix inverse $\mathbf{P}_B[i]$ employs the matrix inversion lemma

$$\mathbf{P}_B[i] = \lambda^{-1}\mathbf{P}_B[i-1] - \lambda^{-1}\mathbf{k}_B[i]\hat{\mathbf{x}}^H[i]\mathbf{P}_B[i-1]. \tag{25}$$

Equations (19)-(25) constitute the second and the third part of the proposed algorithm, which estimate $\bar{\mathbf{w}}_j[i]$ and $\mathbf{f}_j[i]$. The computational complexity of the proposed RLS algorithms is $O(D^2)$ for the estimation of $\bar{\mathbf{w}}_j[i]$, $O((LN_R)^2)$ for the estimation of $\mathbf{S}_{D,j}[i]$ and $O(N_T^2)$ for the estimation of $\mathbf{f}_j[i]$. Because $D << LN_R$, as will be explained in the next section, the overall complexity is in the same order of the conventional full-rank RLS algorithm ($O((LN_R)^2)$) .

VI. SIMULATIONS

In this section, we assess and compare the bit error rate (BER) performance of the adaptive MIMO decision feedback equalization schemes with different estimators designed according to the LS criterion, namely, the full-rank [6], the reduced-rank MWF [19] when the turbo coding and decoding of [28] is removed, and the AVF [21] techniques for the design of the receivers. The proposed adaptive reduced-rank MIMO DFE employs a reduced-rank estimator $\bar{\mathbf{w}}_j[i]$ with D coefficients for the feedforward section, followed by a feedback structure estimator $\mathbf{f}_j[i]$ with full-rank estimators that perform interference cancellation. For all simulations, we use the initial values $\bar{\mathbf{w}}_j[0] = [1, 0, \ldots, 0]$, $\mathbf{f}_j = \mathbf{0}_{N_T \times 1}$, and $\mathbf{S}_{D,j}[0] = [\mathbf{I}_D \ \mathbf{0}_{D\times(JM-D)}]^T$, assume $L = 5$ as an upper bound, use 3-path channels spaced by one symbol and with relative powers taken from complex Gaussian random variables with zero mean and unit variance and QPSK modulation. The channel is static over one packet, we average the experiments over 100 runs and define the signal-to-noise ratio

(SNR) as SNR $= 10\log_{10}\frac{N_T\sigma_x^2}{\sigma^2}$, where σ_x^2 is the variance of the transmitted symbols and σ^2 is the noise variance. The adaptive MIMO DFE employs $N_T = 4$ and $N_R = 8$ in a spatial multiplexing configuration, leading to estimators at the receiver with $LN_R = 40$ elements. The adaptive LS estimators of all methods are trained with 250 symbols and then switched to decision-directed mode.

We consider the BER performance versus the rank D with optimized parameters (forgetting factors $\lambda = 0.998$) for all schemes. The results in Fig. 4 indicate that the best rank for the proposed scheme is $D = 4$ (which will be used in the remaining experiments) and it is close to the optimal MMSE that assumes the knowledge of the channel and the noise variance. Our studies with systems with different sizes show that the optimal rank D does not vary significantly with the system size. It remains in a small range of values and does not scale with system size, which brings considerably faster convergence speed. However, It should also be remarked that the optimal rank D depends on the data record size and other parameters of the systems. In order to tackle this problem, we plan to devise in the future an adaptive rank selection algorithm that will automatically adjust the best rank and will be considered elsewhere.

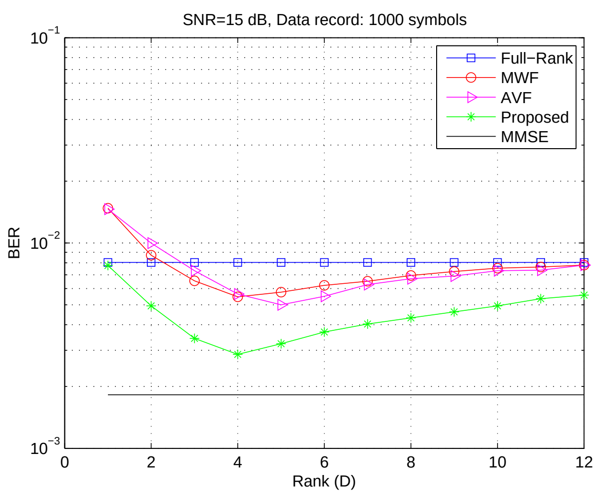

Fig. 4. BER performance versus rank (D).

The BER convergence performance versus number of received symbols is shown in Fig. 5. The results show that the proposed scheme has a significantly faster convergence performance and obtains good gains over the best known schemes. The BER performance versus the signal-to-noise ratio (SNR) is shown in Fig. 6. The plots show that the proposed reduced-rank MIMO equalizer and estimation algorithm have the best performance followed by the AVF, the MWF, and the full-rank estimator.

The advantages of the reduced-rank estimators are due to the reduced amount of training and the relatively short data record (packet size). Therefore, for packets with relatively small size, the faster training of reduced-rank LS estimators will lead to

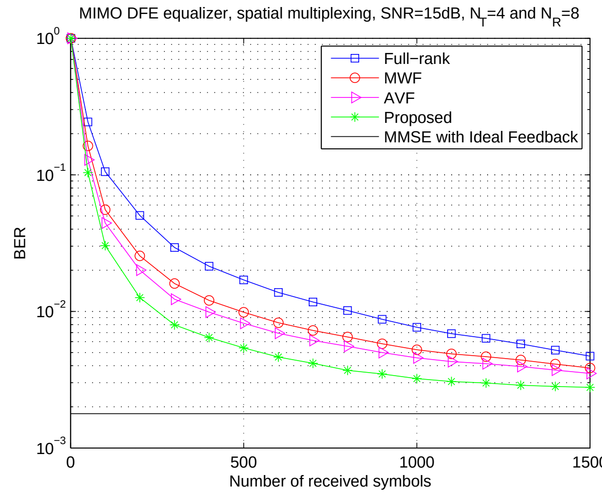

Fig. 5. BER performance versus number of received symbols.

superior BER to conventional full-rank LS estimators. As the length of the packets is increased, the advantages of reduced-rank estimators become less pronounced for training purposes and so become the BER advantages over full-rank estimators. In comparison with the MWF and AVF reduced-rank schemes, the proposed scheme exploits the joint and iterative exchange of information between the projection matrix and the reduced-rank estimators, which leads to better performance.

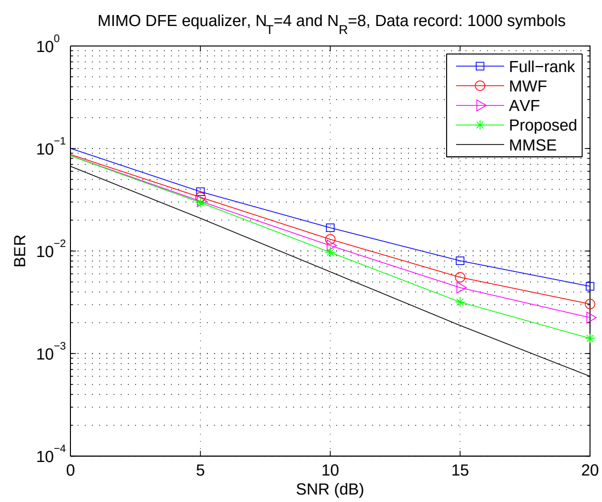

Fig. 6. BER performance versus E_b/N_0 .

VII. Concluding Remarks

This paper proposed an adaptive reduced-rank MIMO decision feedback equalization (DFE) structure based on joint iterative optimization of adaptive estimators. We described LS expressions and efficient RLS algorithms for the design of the proposed MIMO DFE. Simulations for a MIMO equalization application show that the proposed schemes outperforms the

state-of-the-art reduced-rank and the conventional estimation algorithms at about the same complexity. Future work will consider the application of the proposed MIMO DFE equalizers and reduced-rank estimators to realistic MIMO channel models.

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